Quadtree Grammars

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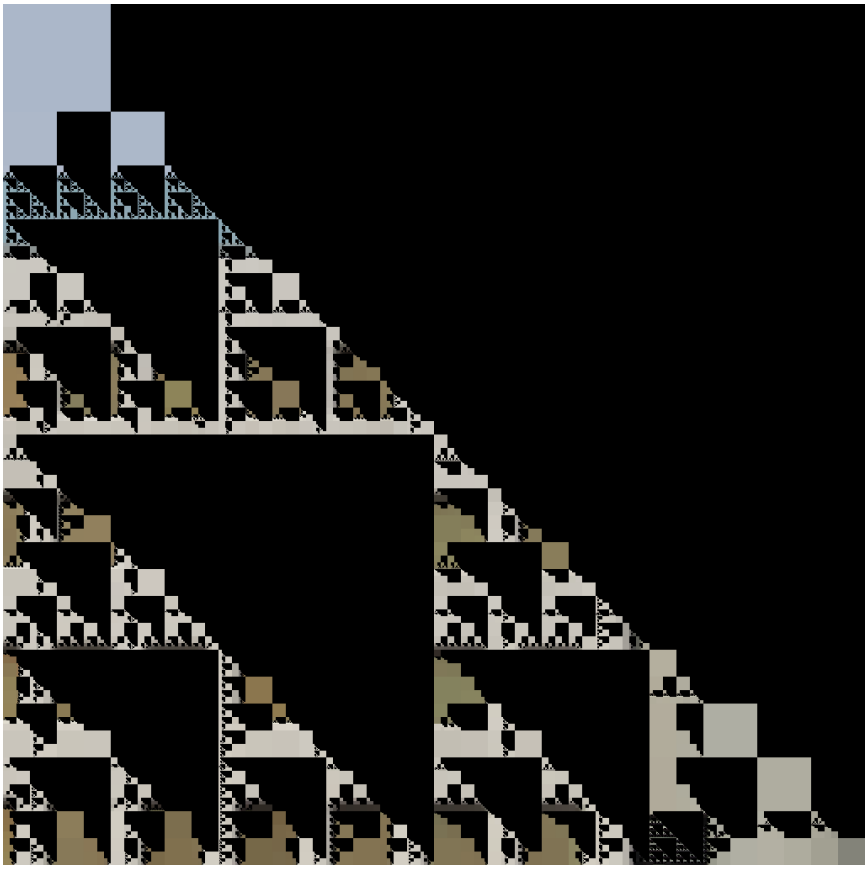
Contents

1 Introduction

A quadtree is a tree where each node has exactly four children. One application is storing 2D data by recursively dividing larger squares into four quadrants (top-left, top-right, bottom-left, bottom-right) and storing these in the tree. Once some condition is fulfilled, e.g. all the pixels in the square have a similar color, the process stops and the square is stored as a leaf.

While experimenting with operations on images compressed this way, a bug in my code produced this image:



As a reference, the original images looks like this:

Why is there a Sierpinsky Triangle hidden in there?

After a bit of debugging, I found out that the upper right corner of each node was not drawn.

The broken images nicely shows how the images is "compressed" by storing parts with the same color as larger squares.

In the upper left, a piece of sky is stored as a big square whereas on the container below the sierpinsky triangles are very visible because the cells of the quadtree are so small.

2 A Framework for Experimentation

What other kinds of patterns and fractals can be generated this way? For further experimentation a bit of tooling is needed.

I'm interested in programming languages, and finding efficient ways to express ideas (see Kolmogorov Complexity) so the solution is obvious: A

small language for defining patterns.

Note: Here, the notion of Kolmogorov complexity — the length of the shortest program necessary to generate some images — is a fuzzy. Which parts of the software should be counted? The rust program? The png library? What about the rust compiler itself?

A pattern is a list of four "pattern elements", one for each quadrant of the quadtree. These element can either be constant colors or references to other patterns.

This forms a subset of the context-free grammars (Chomsky Type-2), the ones with exactly four terminals and non-terminals on the right-hand-side.

In addition to that, each reference to other patterns needs a fallback color to be used once the cells are to small to be divided.

TODO: The code can be found on github TODO: example usage

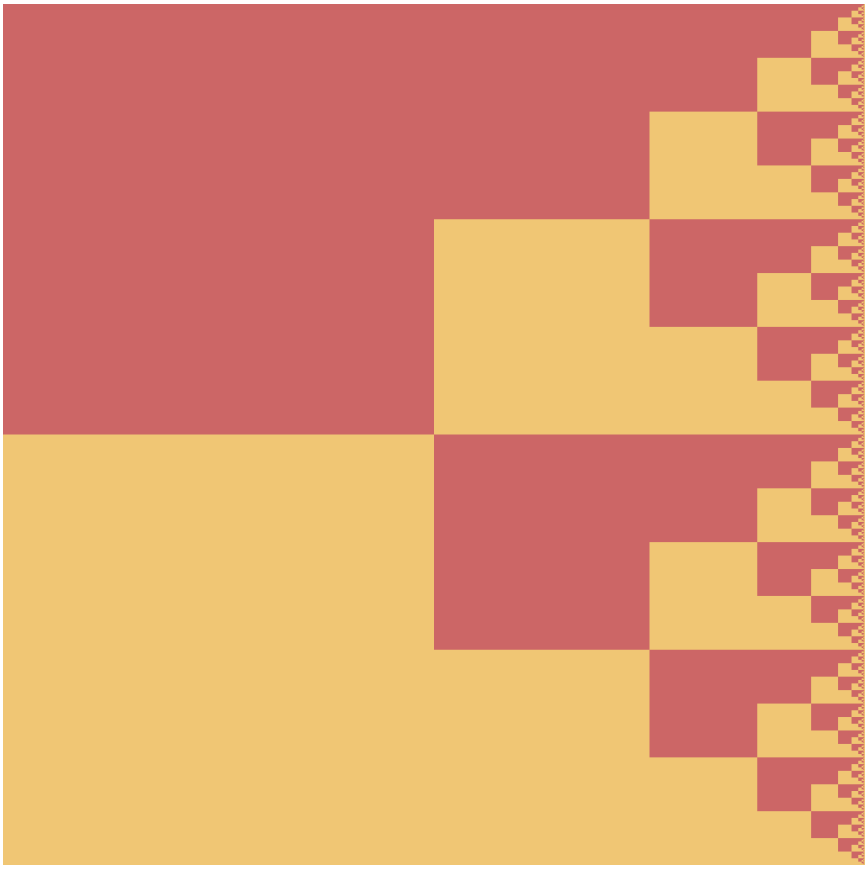
3 Systems With One Rule

Ignoring rotations and different colors, there are only a handful of systems with a single rule.

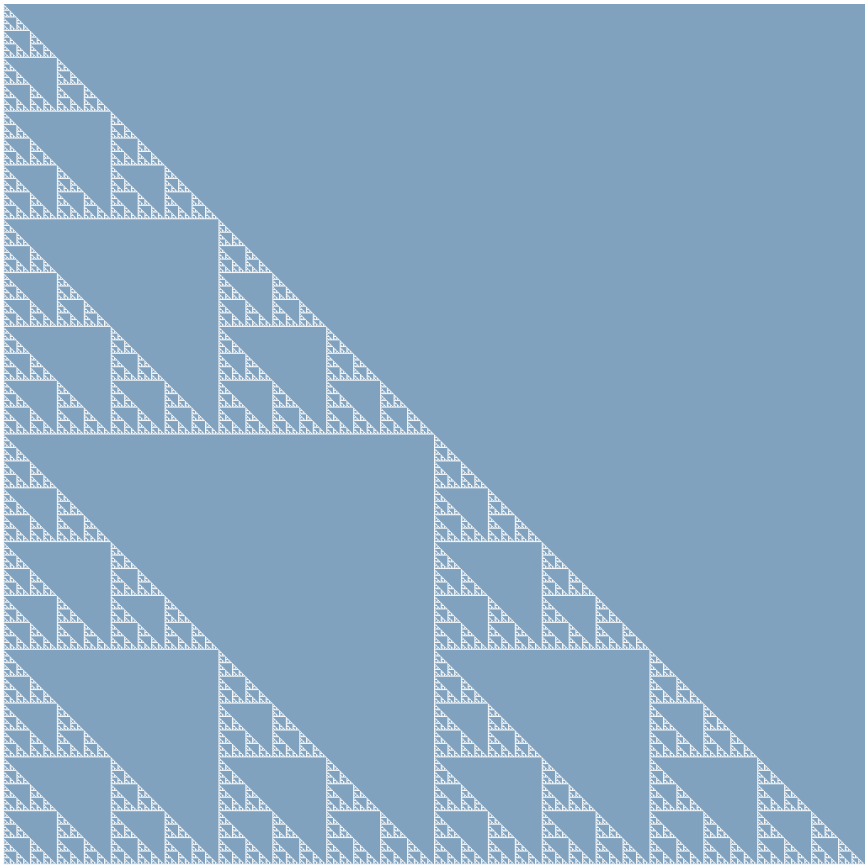
```
S -> white green blue S | white
```

```
S -> S red green S | red
```

```
S -> red S yellow S | yellow
```



```
S -> S blue S S | white
```

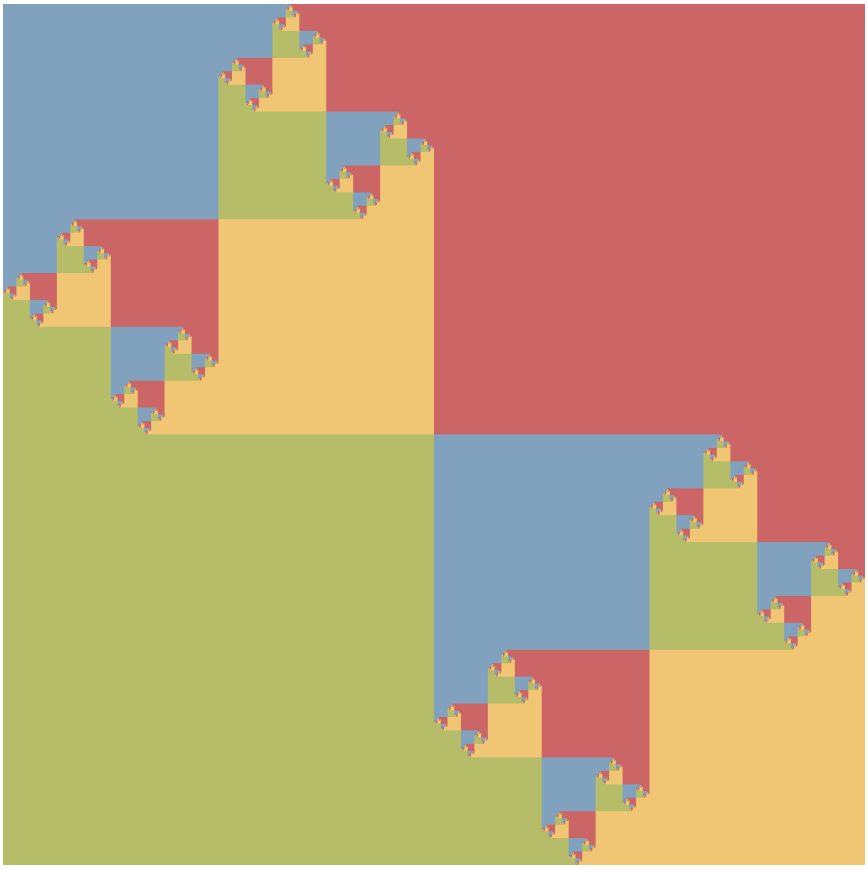


There is the Sierpinski Triangle again.

4 Systems With Two Rules

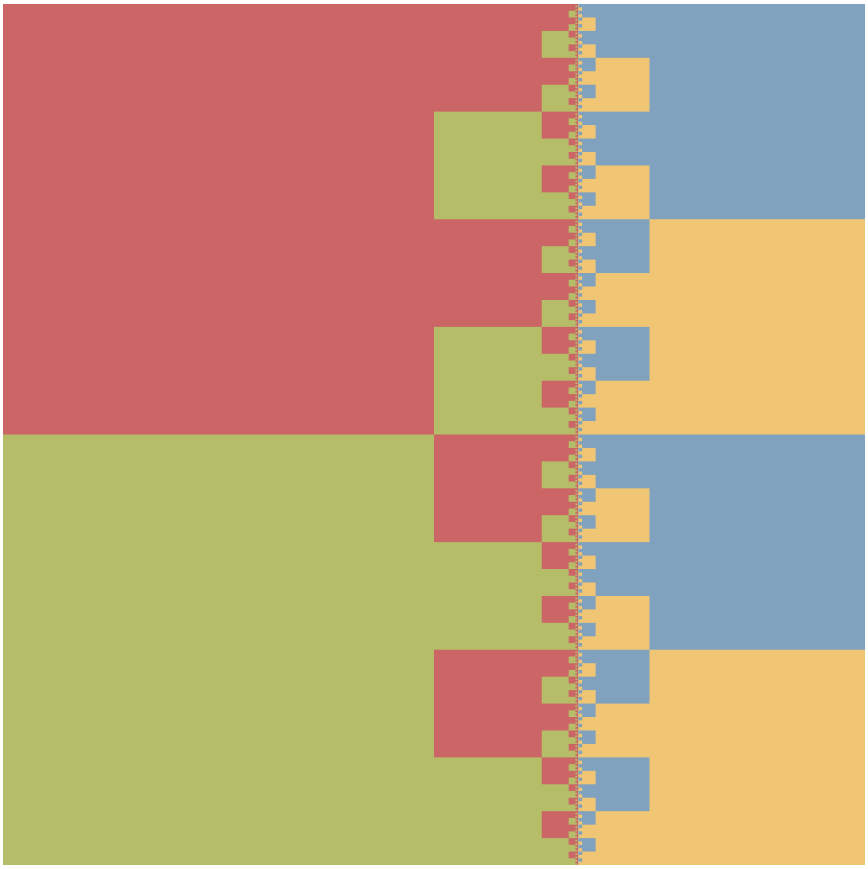
Adding more rules, the number of possible grammars quickly grows.

```
S -> T red green T | red
T -> blue S S yellow | blue
```

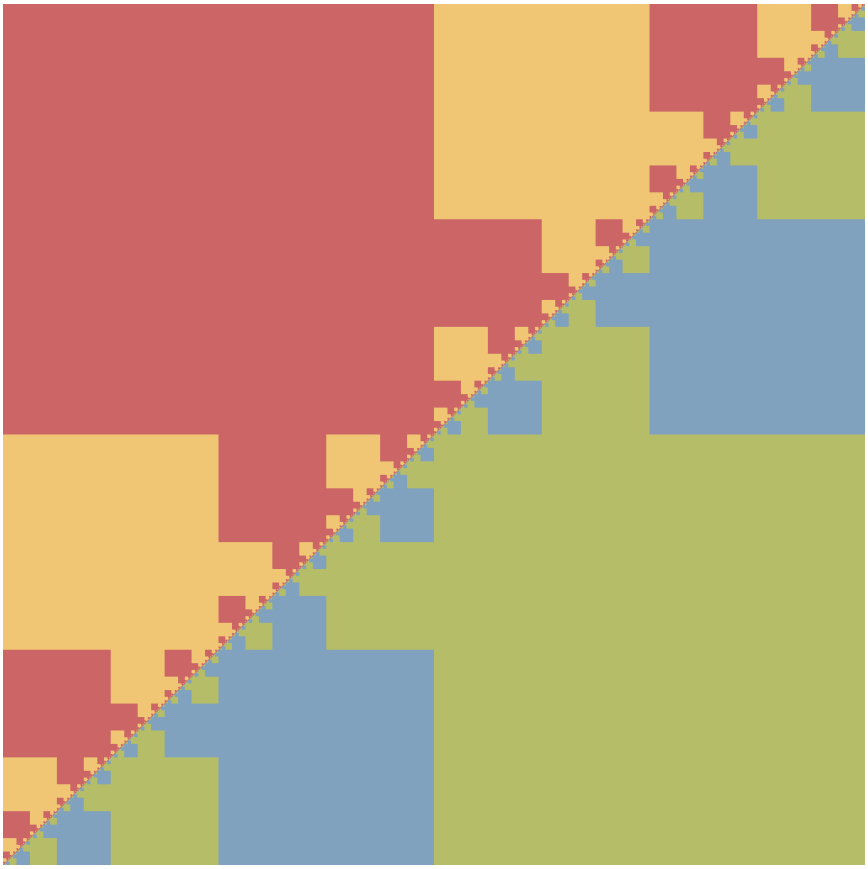


Small changes to the rules lead to very different images.

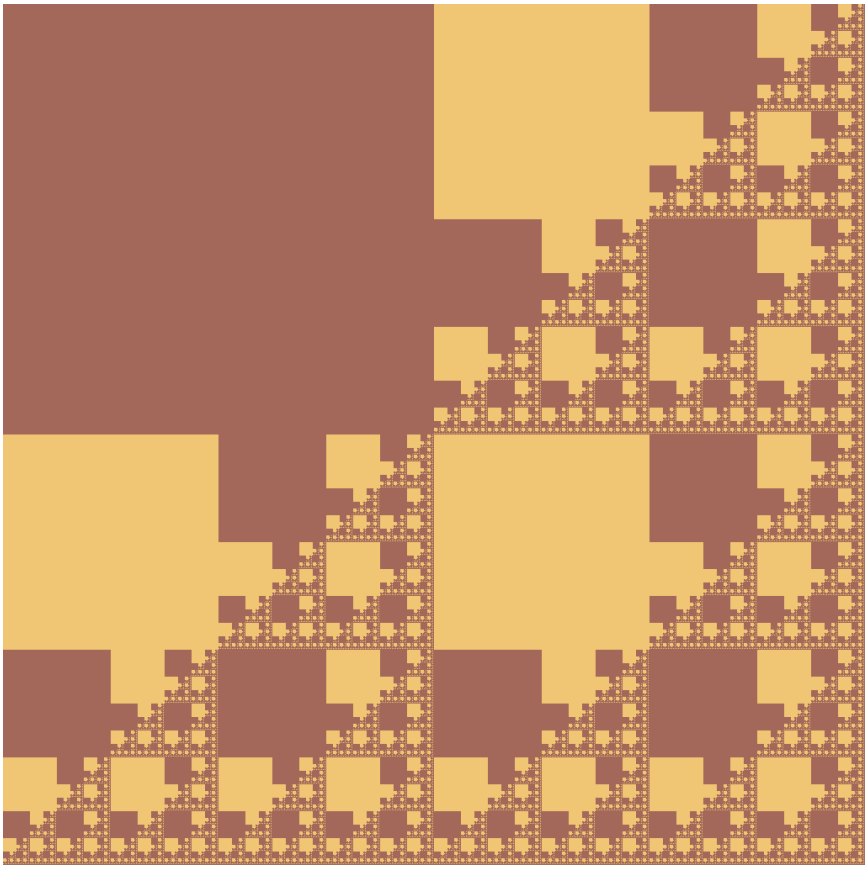
```
S -> red T green T | red
T -> S blue S yellow | blue
```



```
S -> red T T green | red
T -> yellow S S blue | blue
```



```
S -> brown T T T | brown
T -> yellow S S S | yellow
```

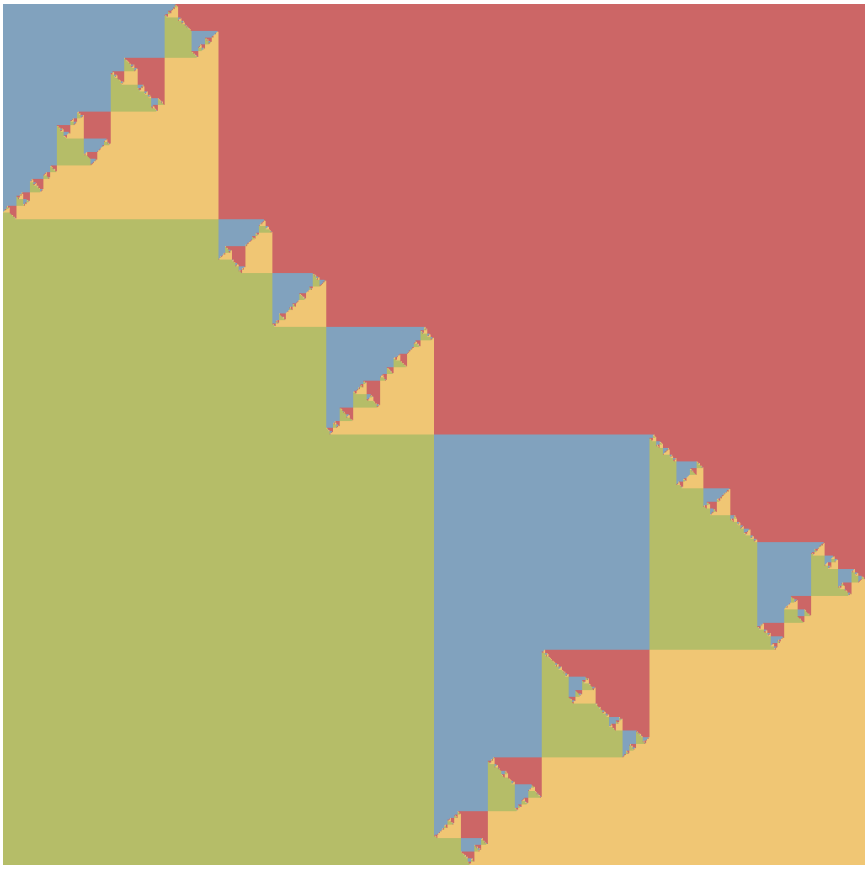


5 Chance and Necessity

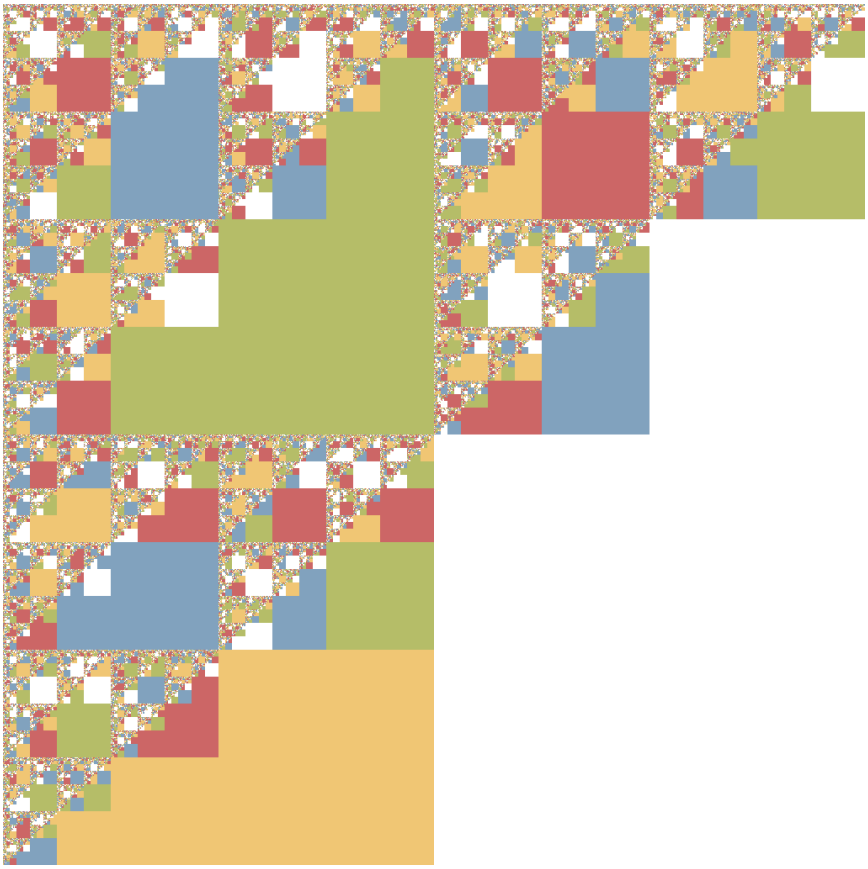
These rule systems are already pretty powerful but I'd like to have a bit more variation.

An easy way to do this is allowing multiple rules with the same left-hand-side (the part before the `->`) and randomly choosing one on each iteration.

```
S -> blue S S yellow | yellow
S -> S red green S | red
```



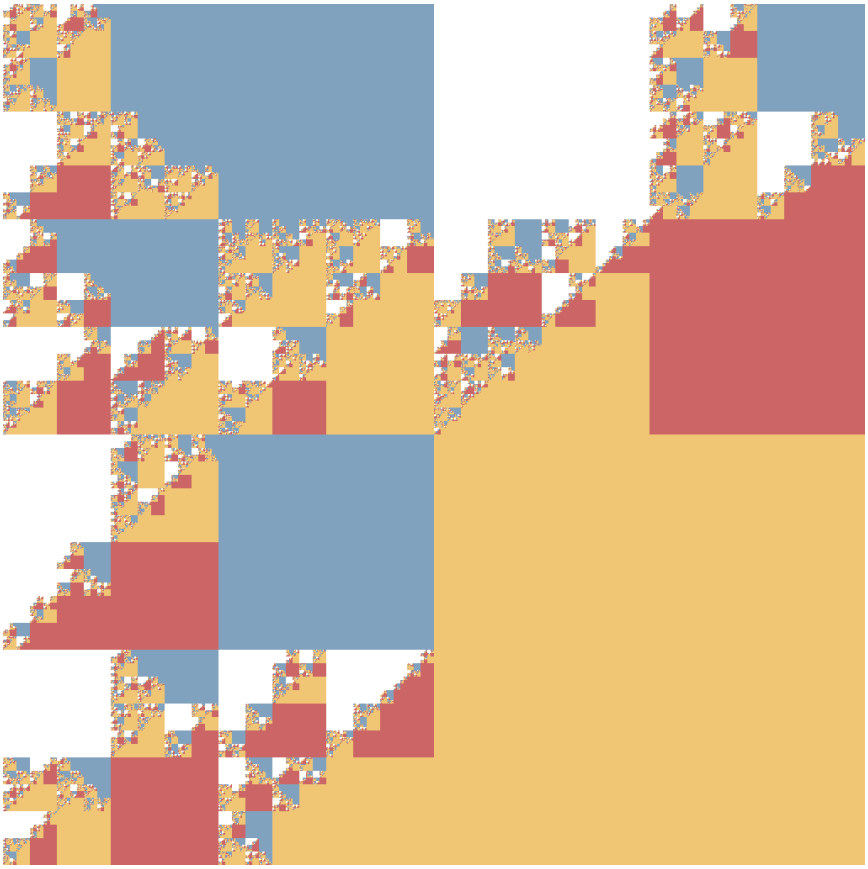
```
S -> S S S white | white
S -> S S S blue | blue
S -> S S S green | green
S -> S S S red | red
S -> S S S yellow | yellow
```



Now the `RuleSet` contains a list of patters for each index and each time one is chosen at random.

language=rust,label= ,caption= ,captionpos=b,numbers=none impl Index<usize> for RuleSet type Output = Pattern; fn index(self, i: usize) -> Pattern let mut rng = self.rng.borrow$_{m}ut(); letj = rng.gen$_{r}ange(0, self.rules[i].len()); self.rules[i][j]$

```
S -> S S S yellow | yellow
S -> S blue S S | blue
S -> white S S red | red
```



6 Letting the Computer do the Work

At this point, the hardest part is finding sets of rules that produce interesting images. Luckily, that's easy to automate by generating random rule sets and choosing the ones that look the best.

When generating a branch of rule, the probability a reference to another rule is chosen is 75%, otherwise a constant color is used.

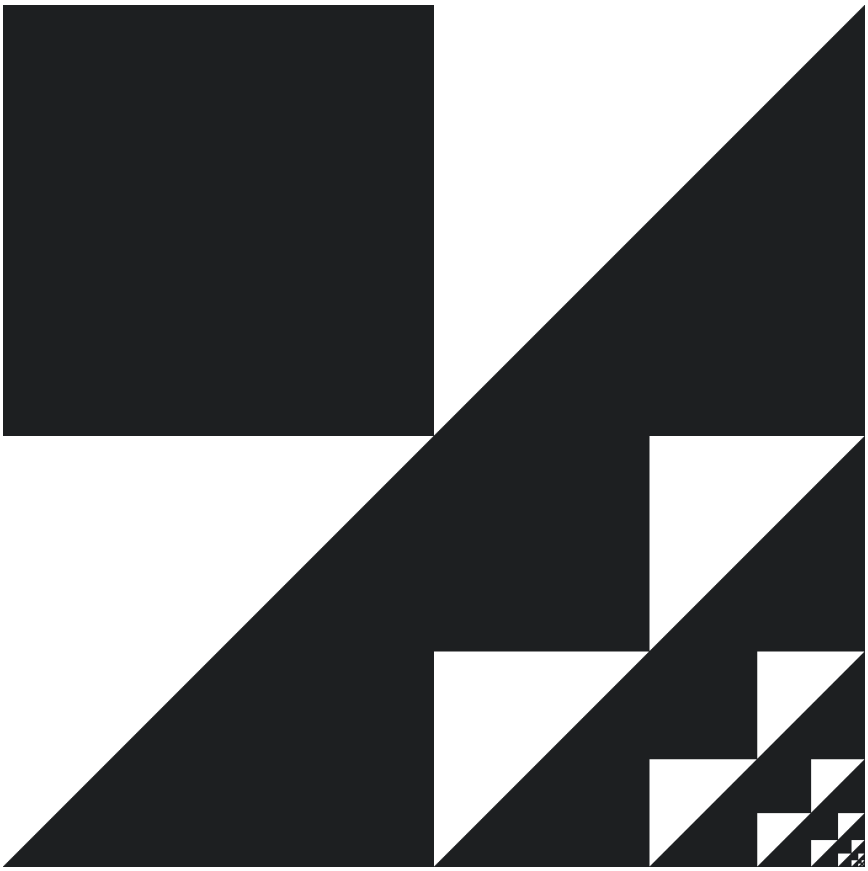
For rules with at least one constant color, the last of them is used as fallback color, otherwise a random one is generated.

Due to the way the systems are generated, there is a chance some of the rules can't be reached from the starting symbol (`S` in the previous sections, `0` for the randomly generated ones).

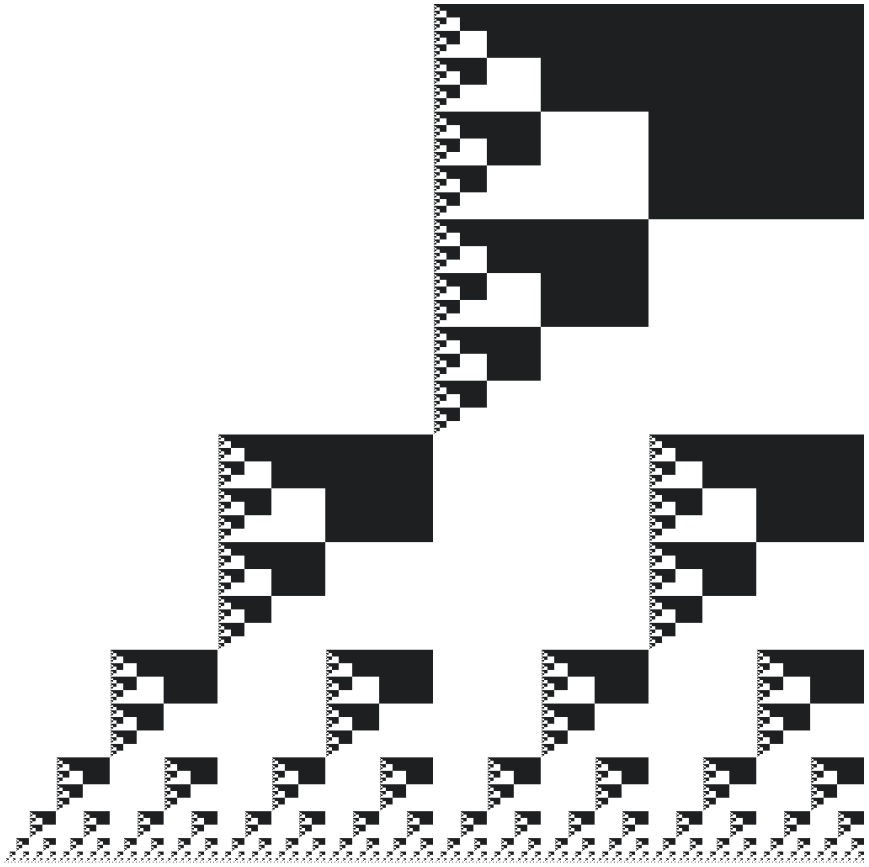
Here are a few of my favorites with different color pallets and parameters.

6.1 Black & White, 2 to 4 rules with 1 branch

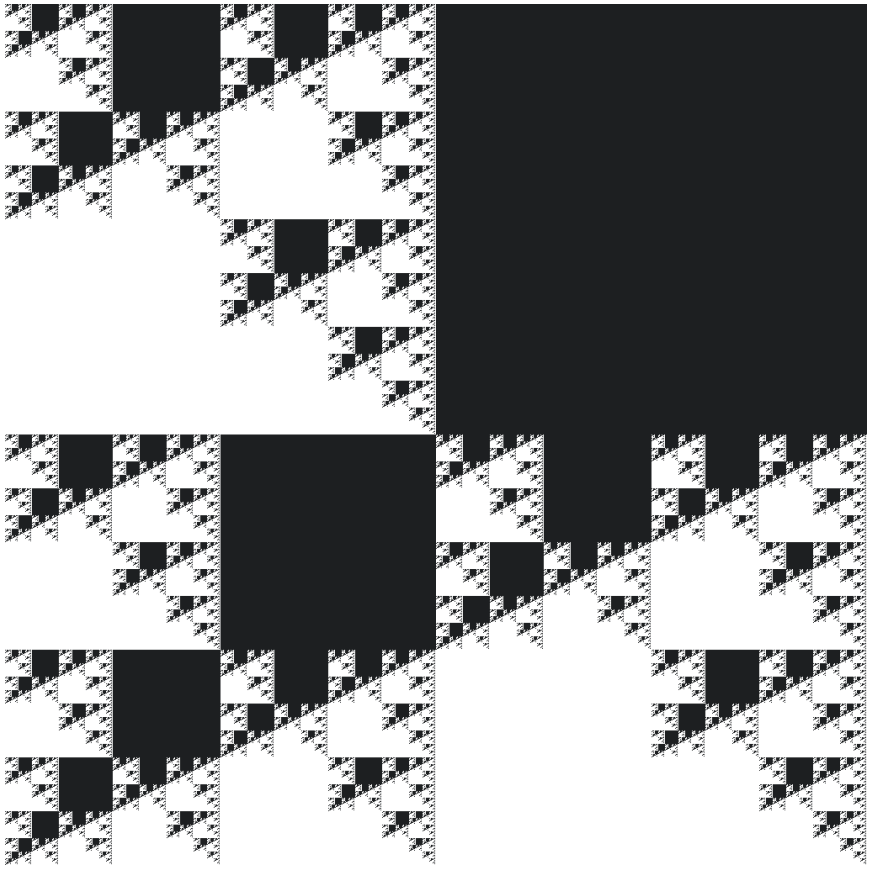
```
0 -> 1 2 2 0 | black
1 -> black 1 1 black | black
2 -> white 2 2 black | black
3 -> 3 1 white 2 | white
```



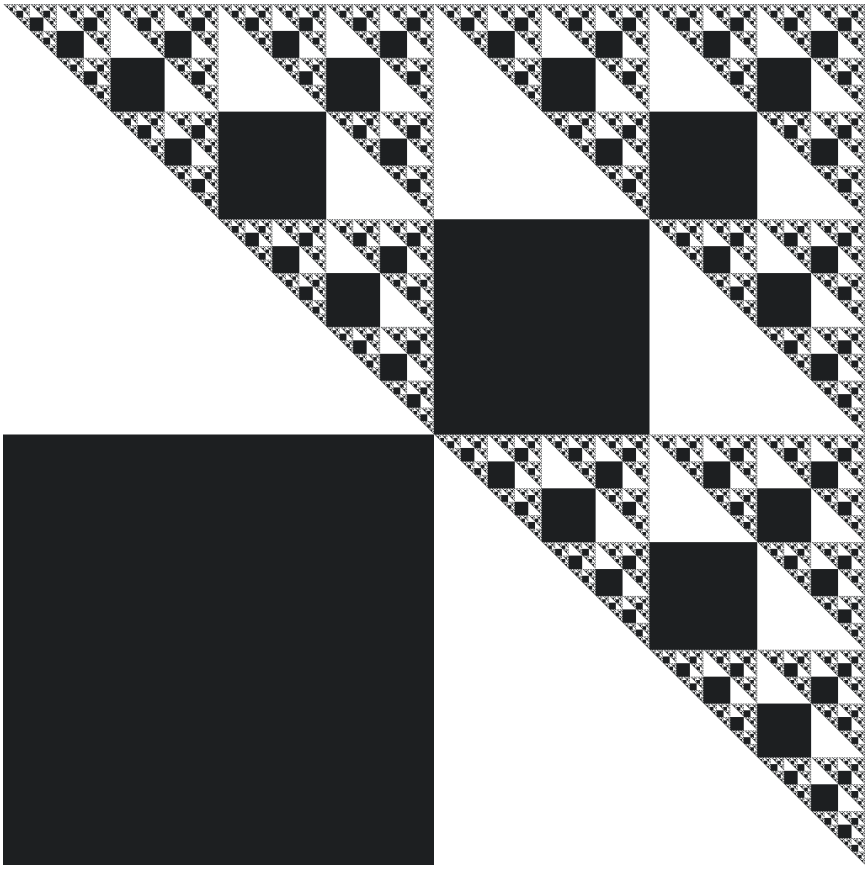
```
0 -> white 1 0 0 | white
1 -> 1 black 1 white | white
```



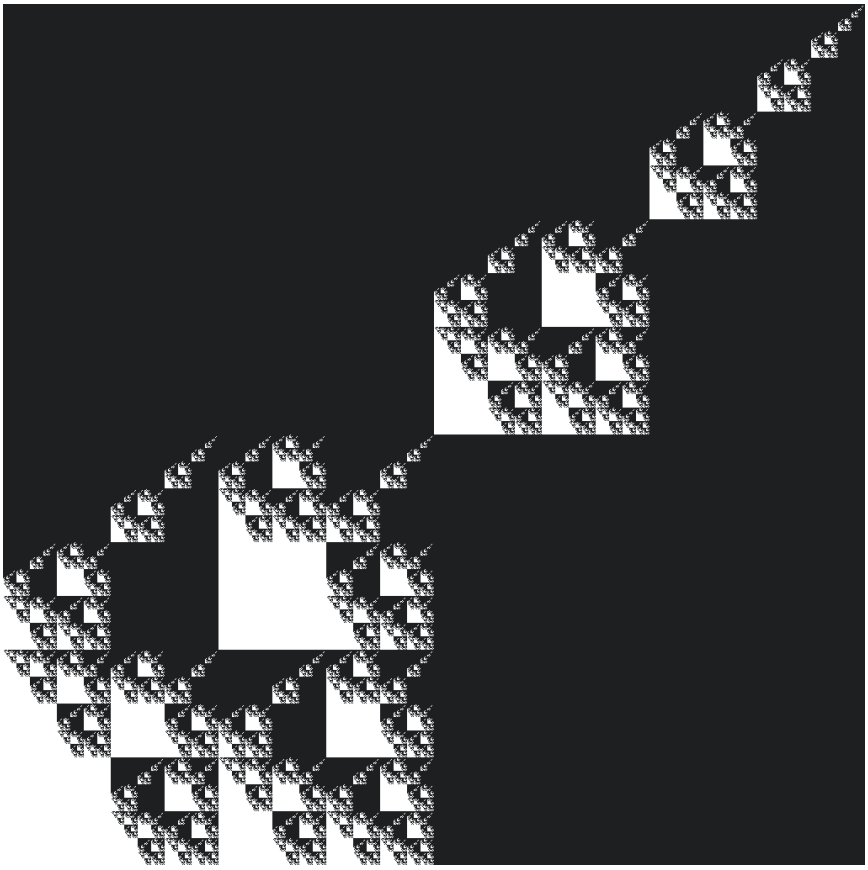
```
0 -> 1 black 0 1 | black
1 -> 0 1 white 1 | white
```



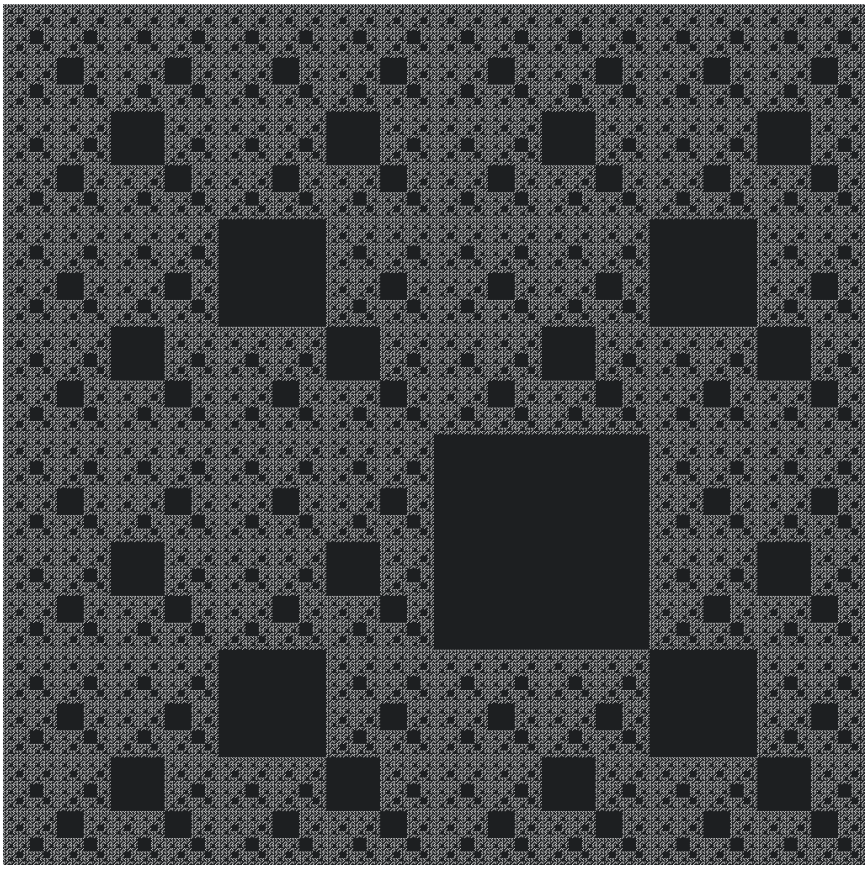
```
0 -> 1 0 black 1 | black
1 -> 1 0 white 1 | white
```



```
0 -> black 0 2 black | black
1 -> 2 0 white 2 | white
2 -> 0 1 3 2 | black
3 -> 3 1 white 2 | white
```

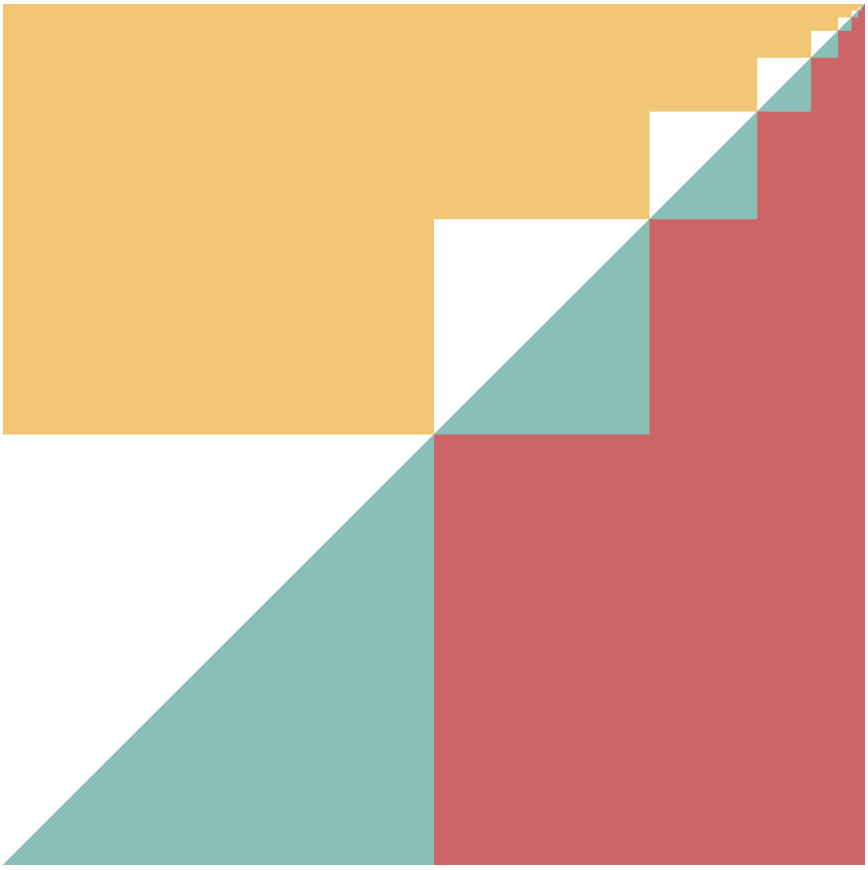


```
0 -> 2 0 2 1 | white
1 -> black 2 2 1 | black
2 -> 2 0 0 1 | black
```

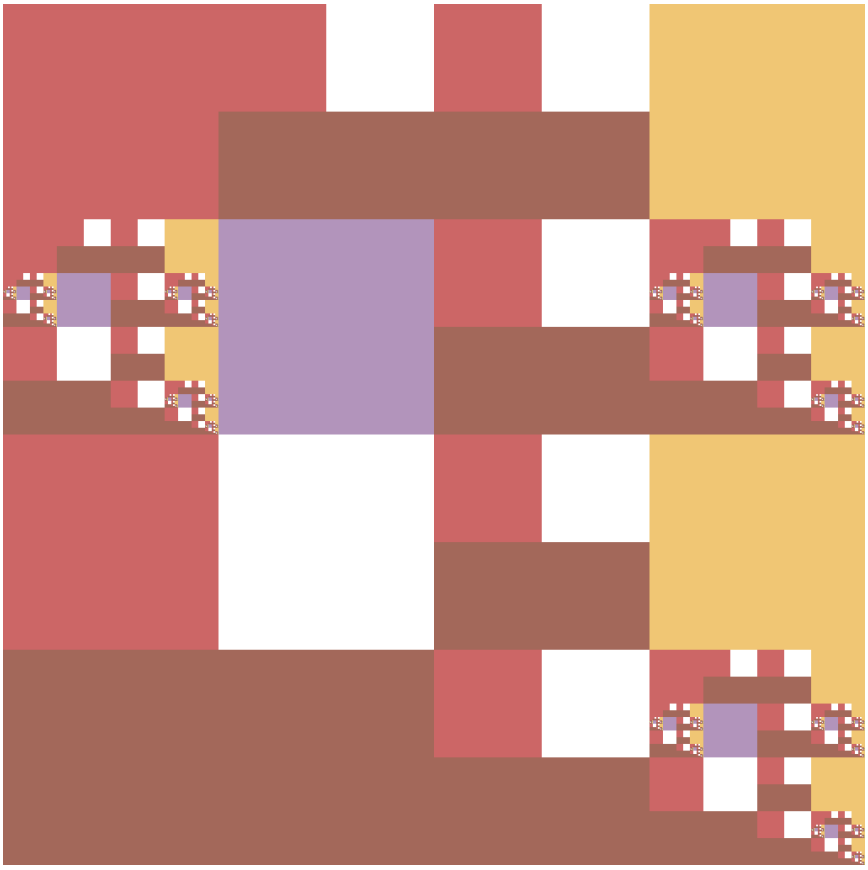


6.2 base16 colors + white, 2 to 4 rules with 1 branch

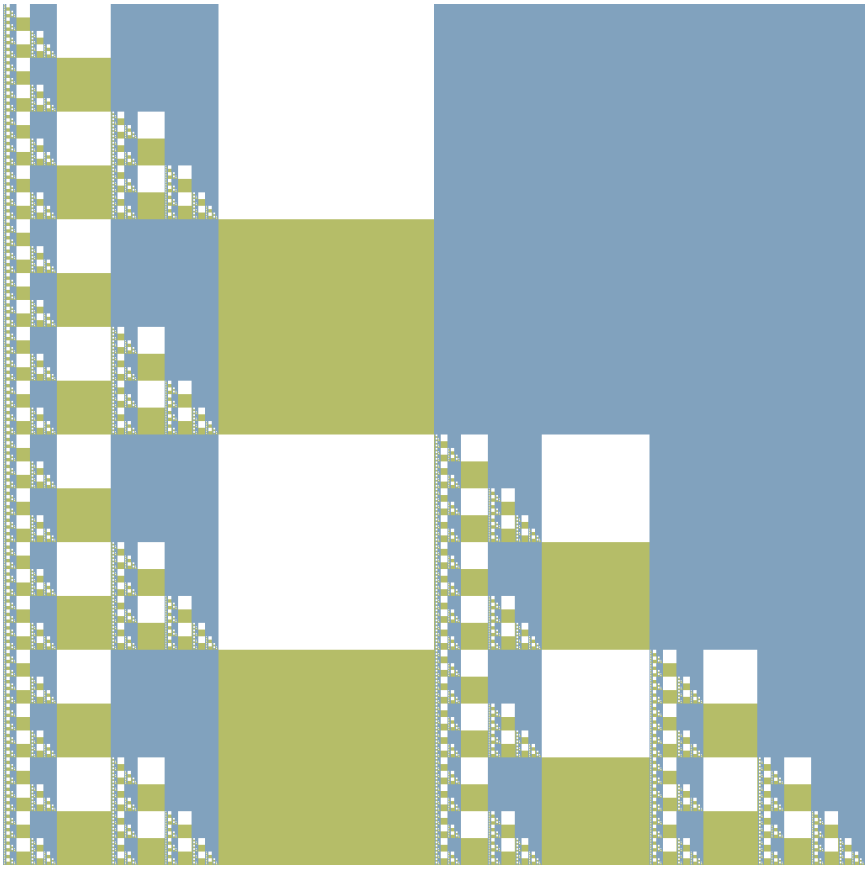
```
0 -> yellow 0 1 red | red
1 -> white 1 1 cyan | cyan
```



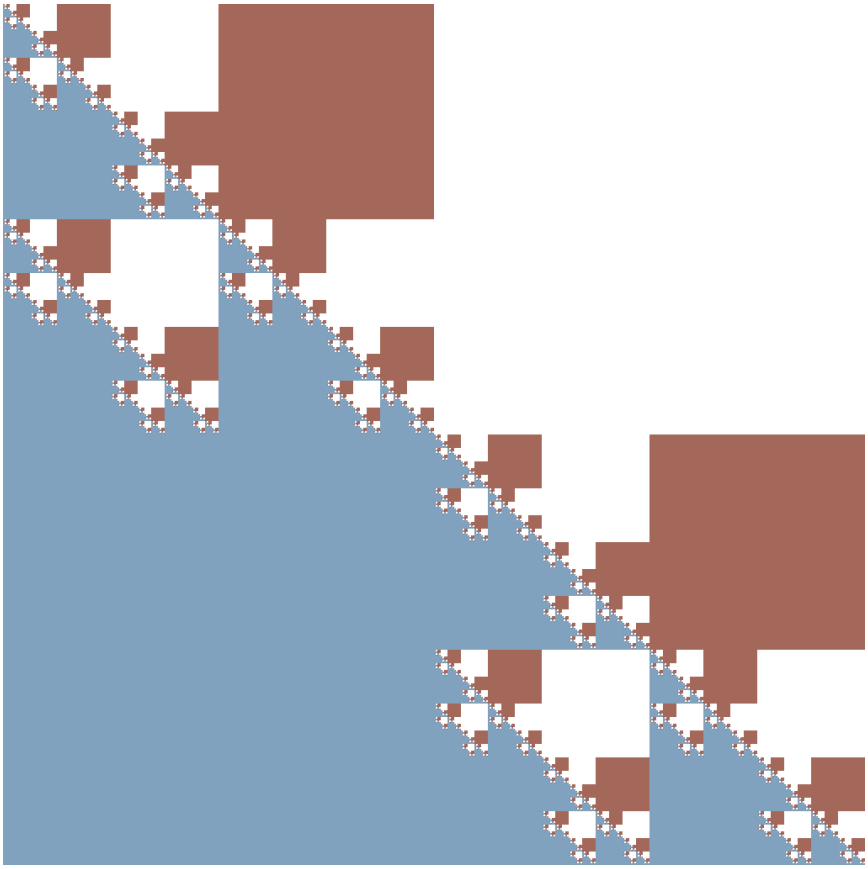
```
0 -> 1 2 3 2 | green
1 -> red 3 0 purple | purple
2 -> 3 yellow 3 0 | yellow
3 -> red white brown brown | brown
```



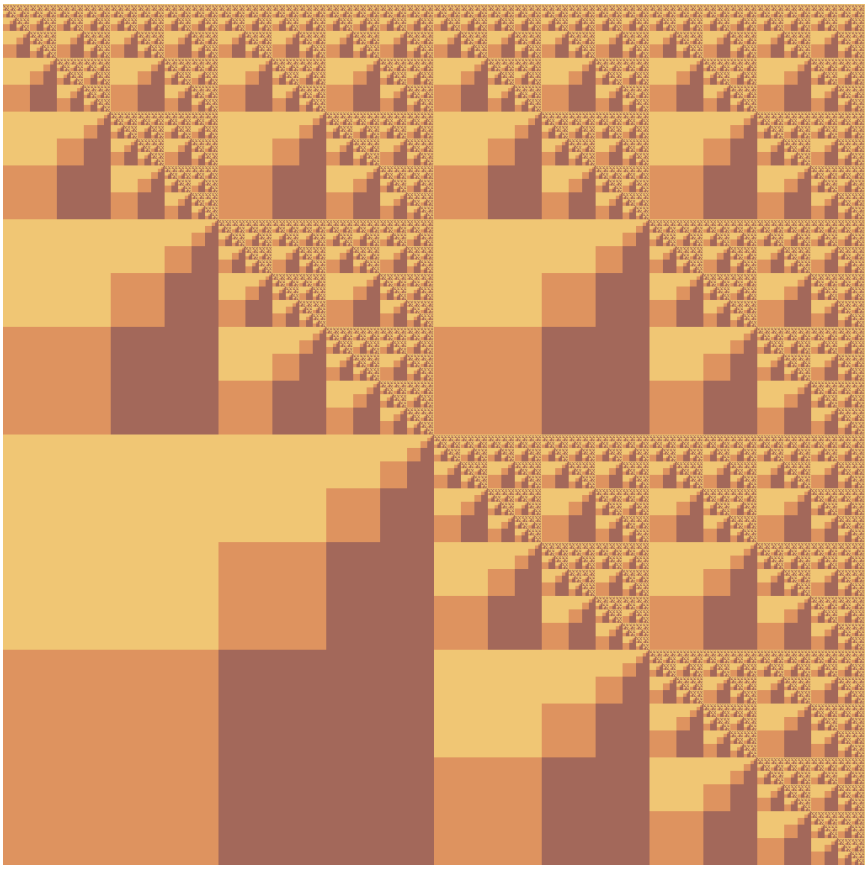
```
0 -> 1 blue 1 0 | blue
1 -> 0 white 0 green | green
```



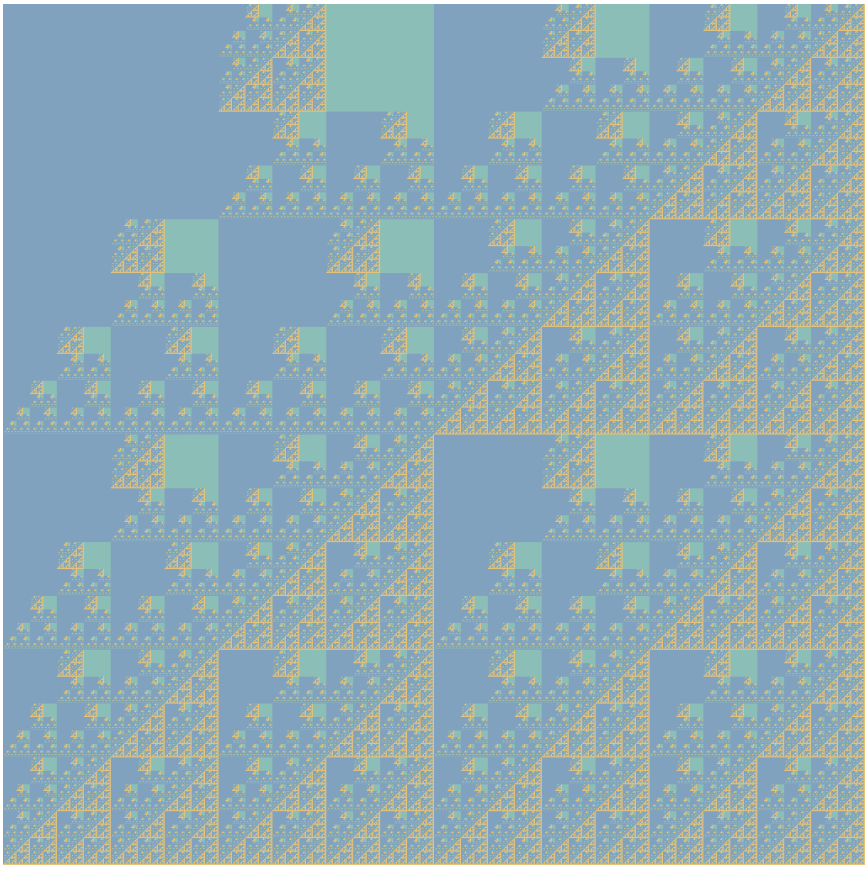
```
0 -> 1 white blue 1 | blue
1 -> 0 brown 0 0 | brown
2 -> 0 2 yellow 1 | yellow
```



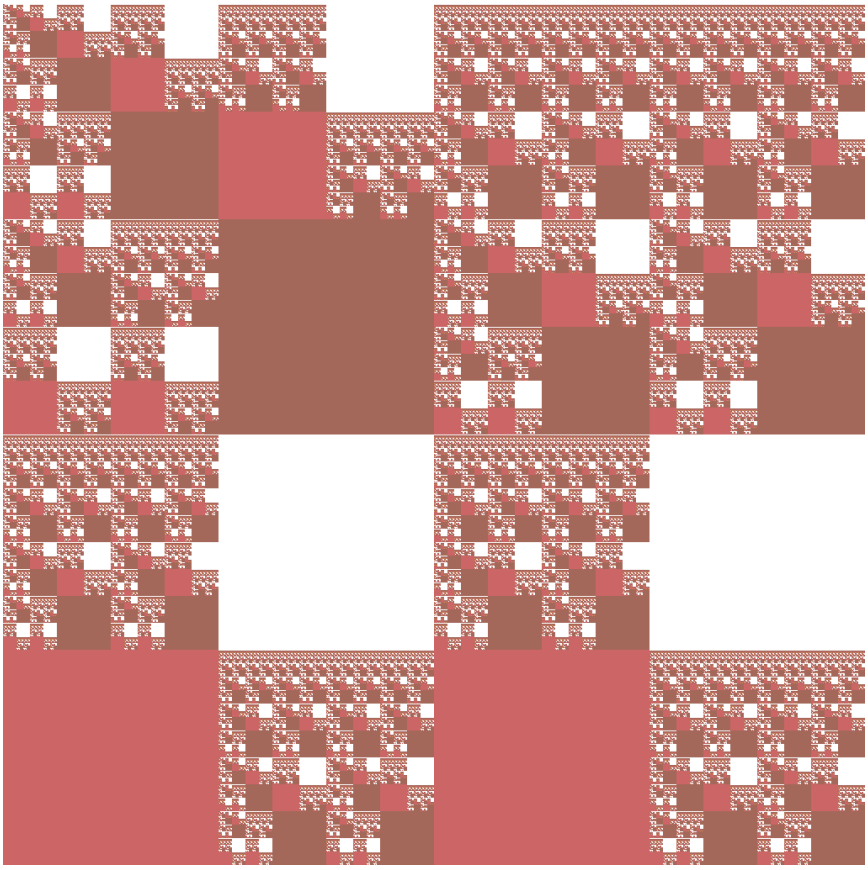
```
0 -> 0 0 1 0 | yellow
1 -> yellow 1 orange brown | brown
```



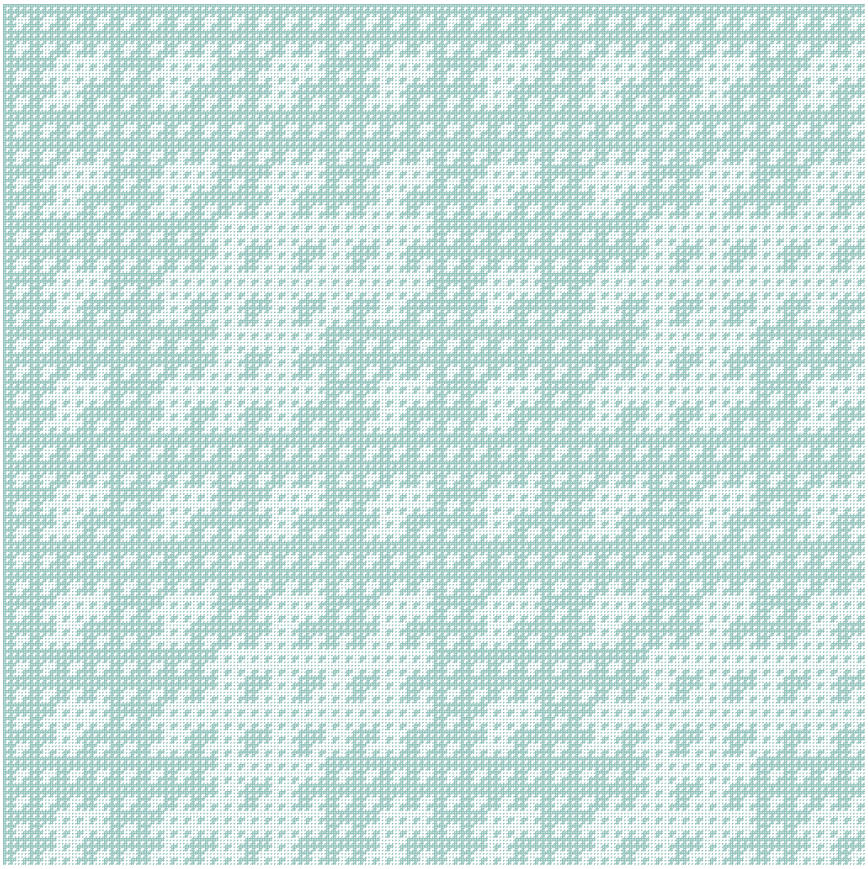
```
0 -> 1 0 0 0 | yellow
1 -> blue 2 1 1 | blue
2 -> 0 cyan 1 1 | cyan
```



```
0 -> 1 3 2 2 | orange
1 -> 1 2 0 brown | brown
2 -> 3 white red 3 | red
3 -> 3 3 1 1 | white
```

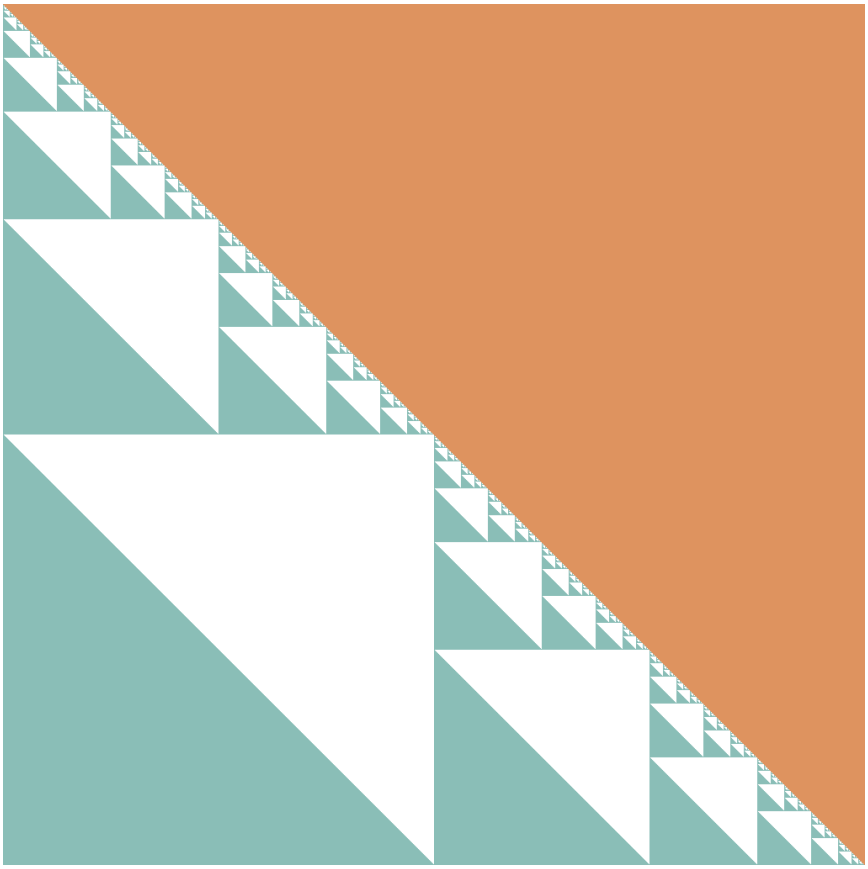


```
0 -> 1 1 1 1 | cyan
1 -> 0 0 0 1 | white
```

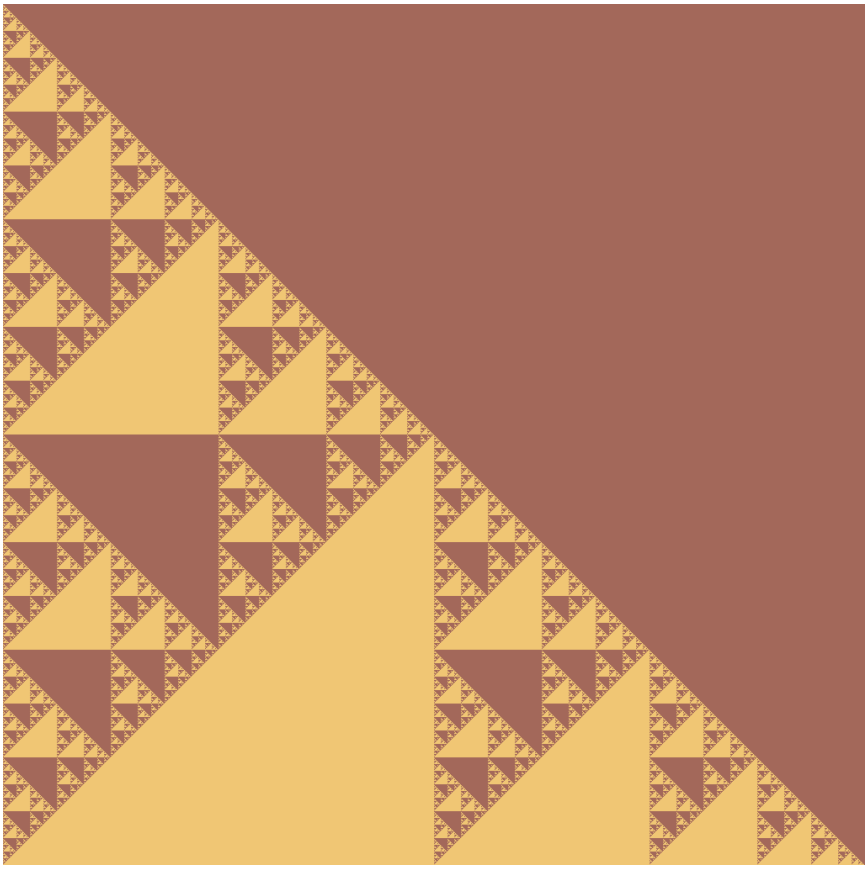


6.3 base16 colors + white, 2 to 4 rules with 1 or 2 branches

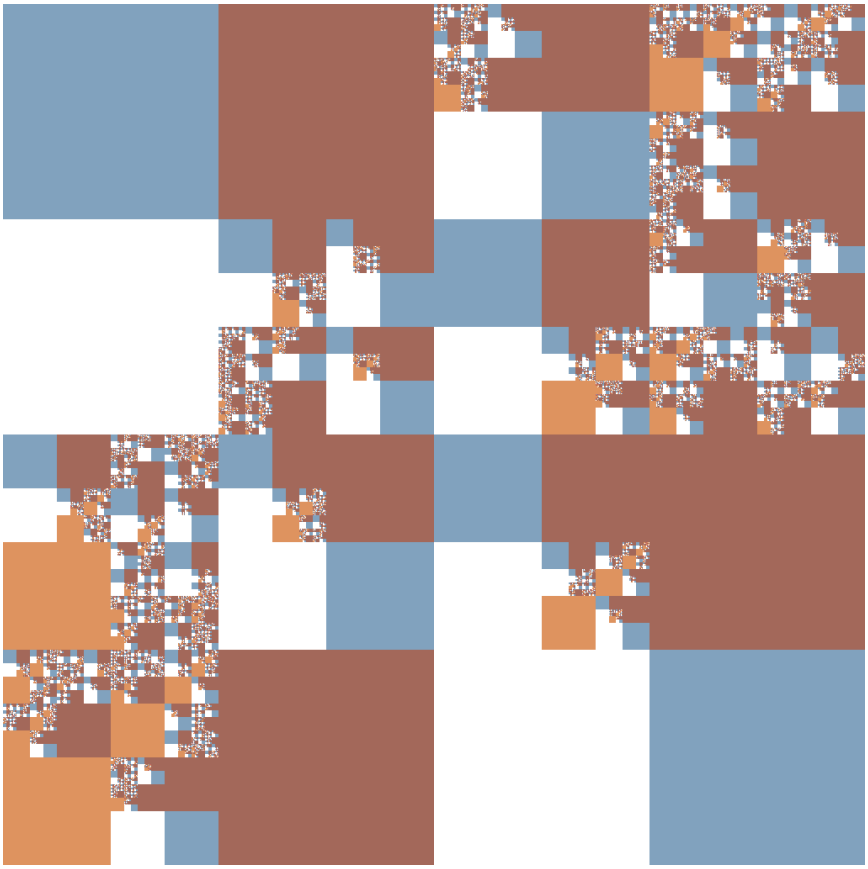
```
0 -> 0 orange 1 0 | orange
1 -> 1 white cyan 1 | cyan
```



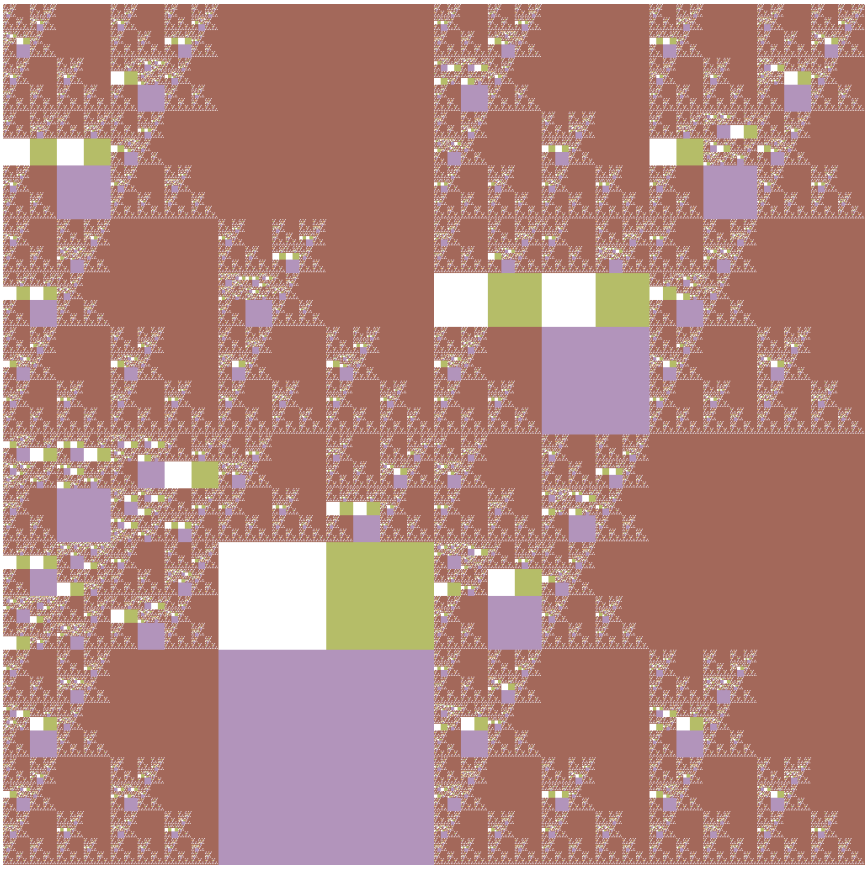
```
0 -> 0 brown 1 0 | brown
1 -> 0 1 1 yellow | yellow
```



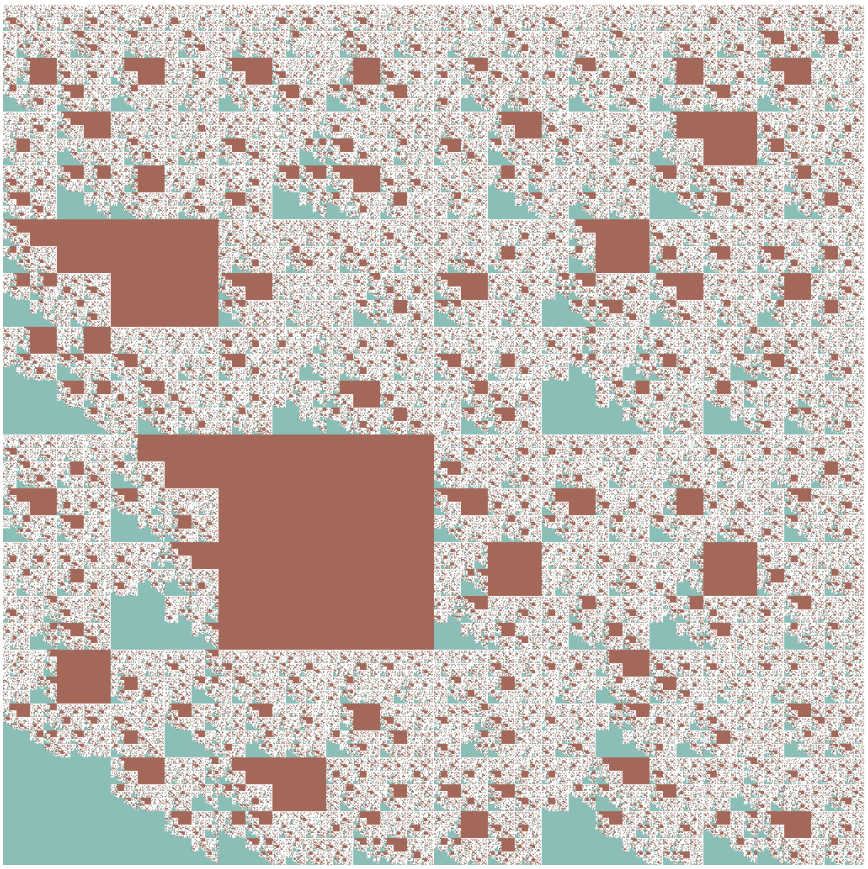
```
0 -> 1 2 1 2 | red
0 -> 1 0 orange 2 | orange
1 -> blue brown white 0 | white
1 -> 0 2 0 brown | brown
2 -> 2 1 1 2 | orange
2 -> 1 brown white blue | blue
```



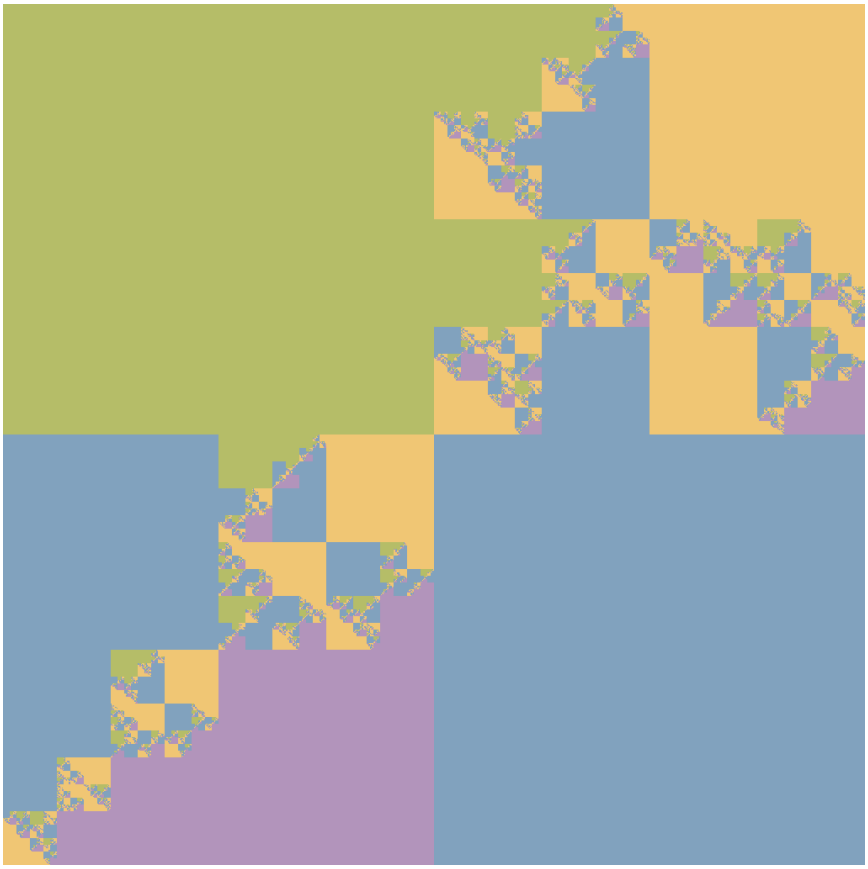
```
0 -> 1 0 2 1 | white
1 -> 0 brown 1 1 | brown
2 -> 3 3 1 purple | purple
3 -> 2 3 3 0 | yellow
3 -> 1 0 white green | green
```



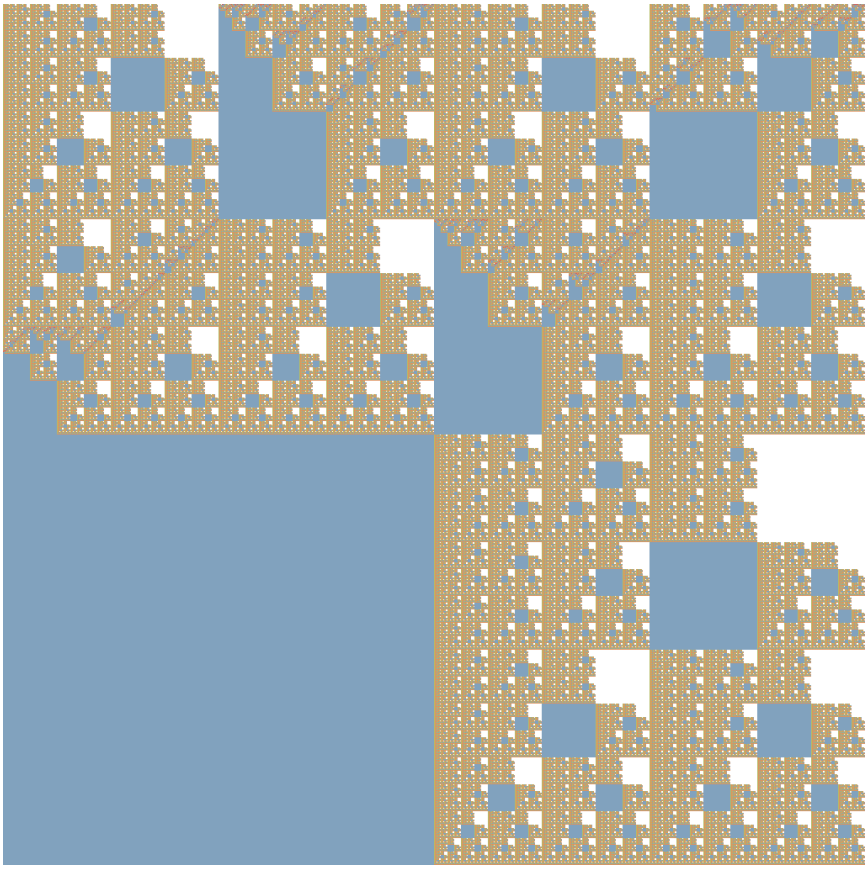
```
0 -> 0 0 1 0 | white
1 -> 0 1 0 2 | white
1 -> 1 brown 2 0 | brown
2 -> 1 1 cyan 2 | cyan
```



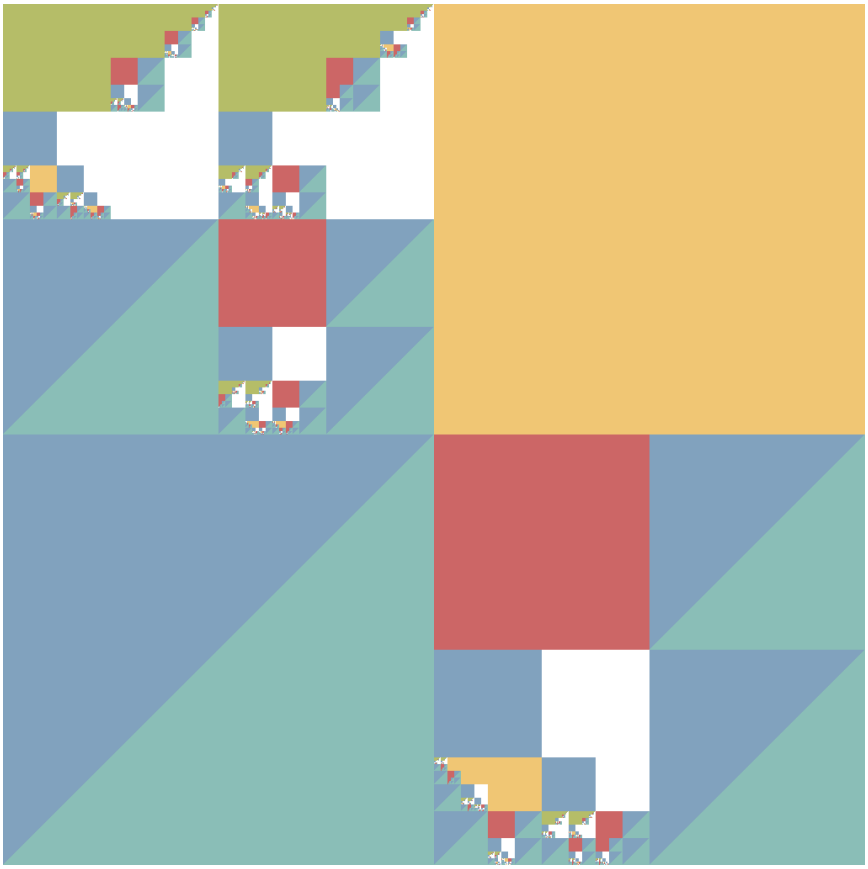
```
0 -> green 0 1 blue | blue
0 -> 0 yellow 0 1 | yellow
1 -> blue 0 1 purple | purple
1 -> 1 0 yellow 1 | yellow
```



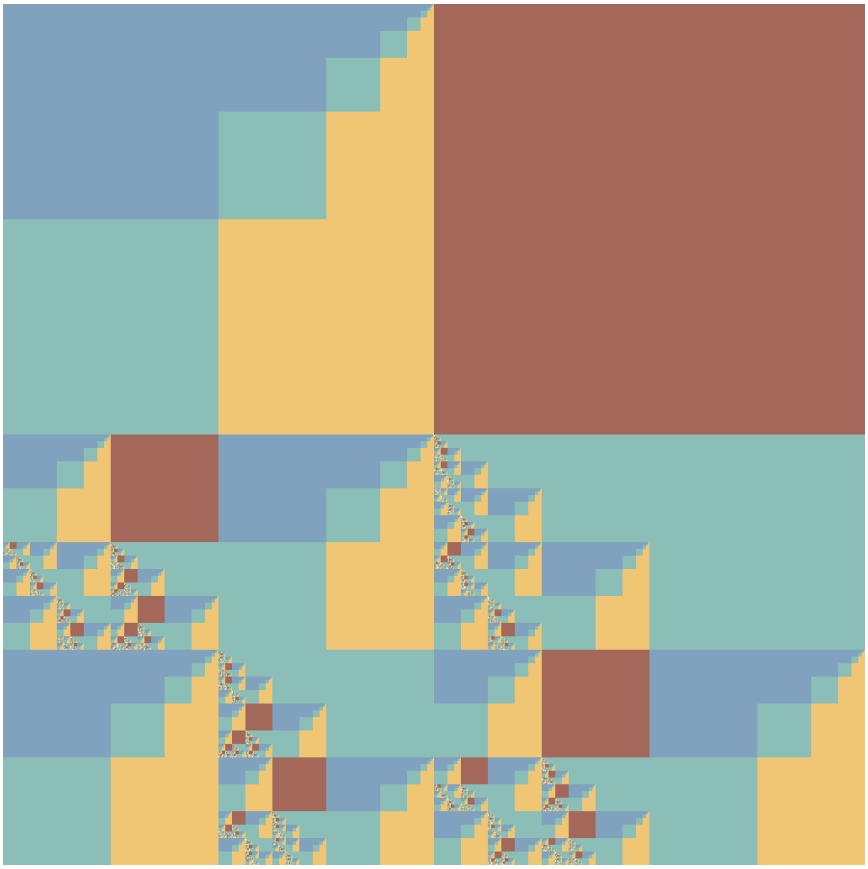
```
0 -> 2 0 0 2 | red
0 -> 0 0 blue 2 | blue
1 -> 1 2 1 2 | green
2 -> 1 3 2 2 | orange
3 -> 1 white blue 2 | blue
```



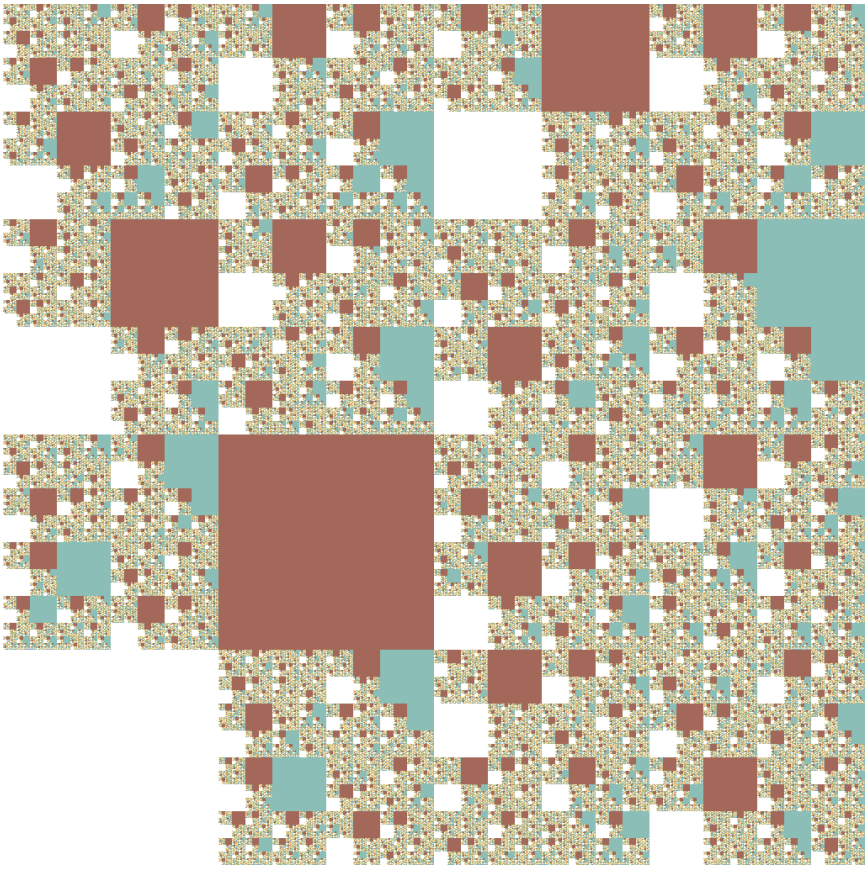
```
0 -> 1 1 3 2 | blue
0 -> 0 yellow 3 2 | yellow
1 -> green 1 2 white | white
2 -> red 3 2 3 | red
2 -> blue white 0 2 | white
3 -> blue 3 3 cyan | cyan
```



```
0 -> 0 2 2 1 | white
0 -> 2 brown 0 1 | brown
1 -> 1 cyan 0 2 | cyan
2 -> blue 2 cyan yellow | yellow
```

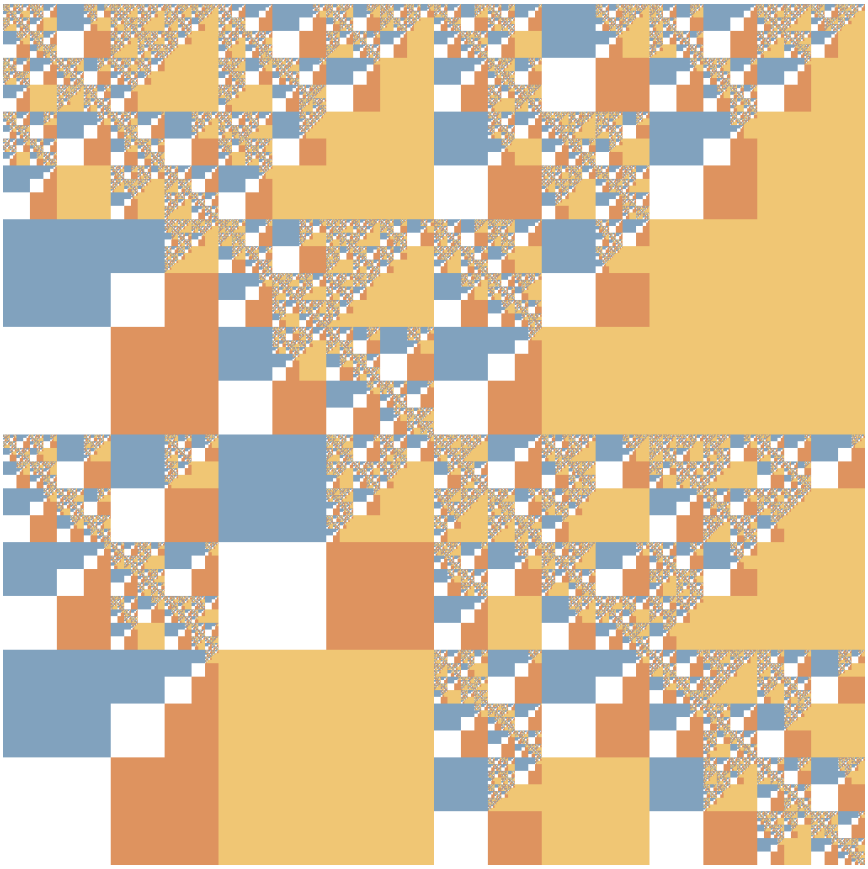


```
0 -> 0 1 2 1 | green
1 -> 2 1 0 3 | yellow
1 -> 0 3 3 0 | yellow
2 -> 1 brown white 1 | white
3 -> 2 cyan 3 3 | cyan
3 -> 2 1 1 3 | blue
```

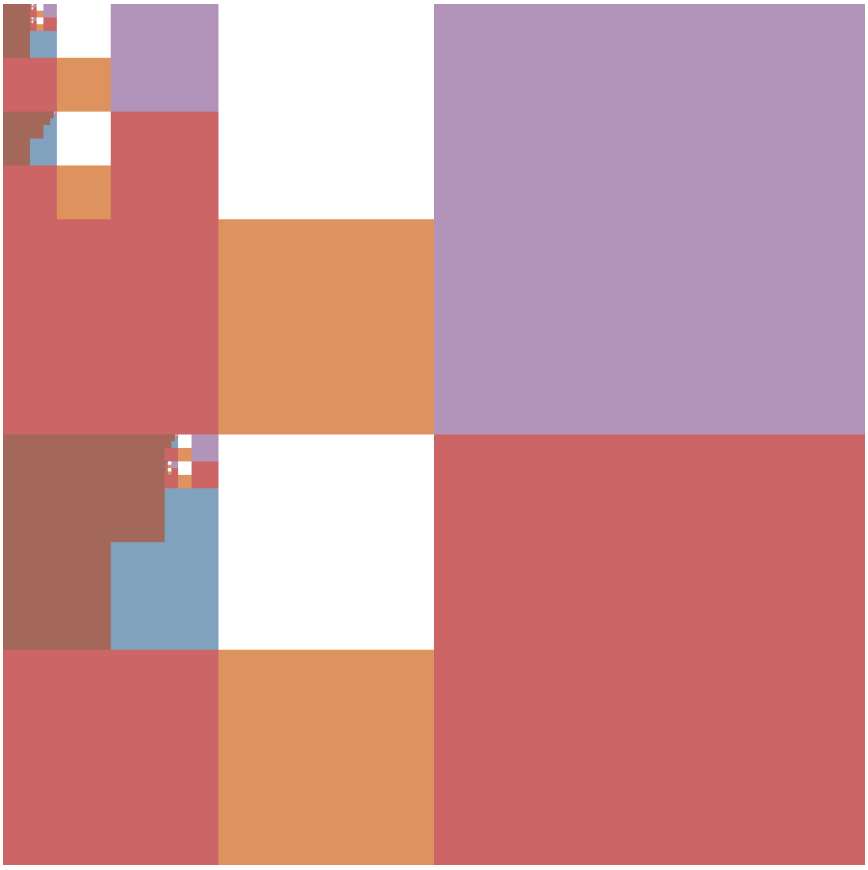


```
0 -> yellow 3 1 2 | yellow
1 -> white 3 white 3 | white
1 -> 0 0 3 3 | yellow
2 -> 1 1 3 brown | brown
3 -> 0 1 yellow 1 | yellow
```

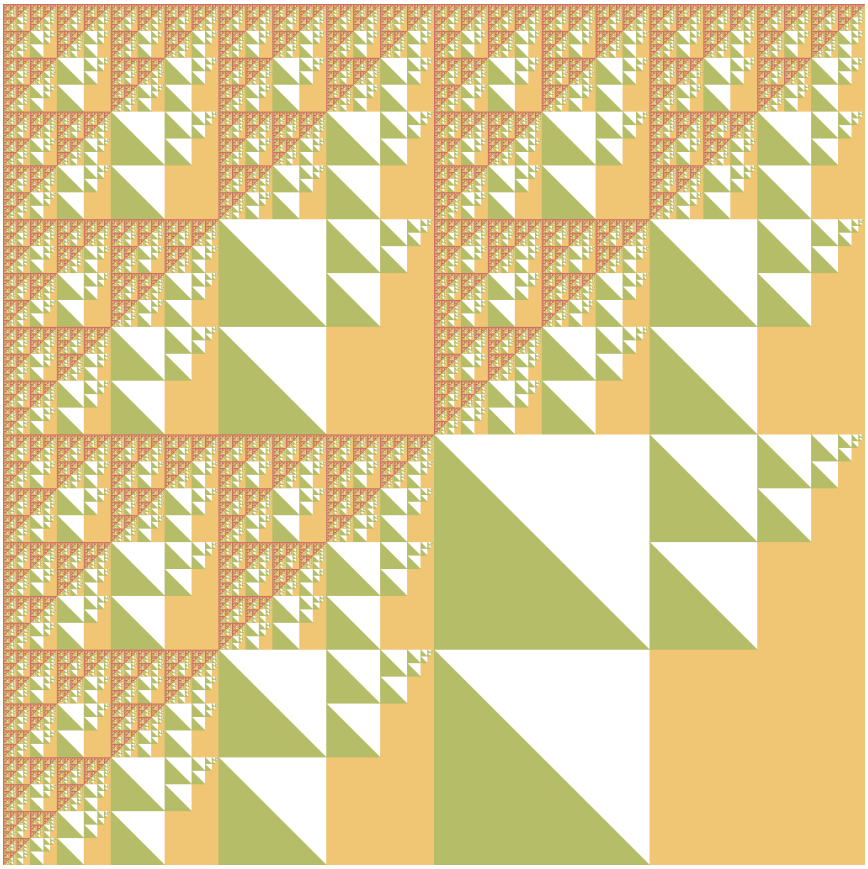
```
0 -> 0 1 1 0 | blue
1 -> blue 1 white orange | orange
1 -> 0 1 1 yellow | yellow
```



```
0 -> brown 0 brown blue | blue
0 -> 1 purple 1 red | red
1 -> 0 white red orange | orange
2 -> 0 2 1 2 | orange
2 -> red blue 1 0 | blue
```



```
0 -> 0 0 0 1 | red
1 -> 2 1 2 yellow | yellow
2 -> 2 white green 2 | green
```



7 Conclusion

Looking back at my selection from the randomly generated rule sets, the sweet spot seems to be systems with two or three rules, beyond that, the images become to "chaotic".

It should be possible to define a taxonomy of patterns based patterns that show up often, e.g. Sierpinski Triangles, diagonal divisions and recursive binary partitions, but that has to wait for another time.

I'm working on a gallery page with a larger number of images for random rule sets and the source code will be uploaded to github soon.

Another direction to take this would be using simple black and white patterns, shapes and symbols so that the results can be plotted with a pen plotter or using octrees to generate random fractal isometric structures.

If you have any questions about this process, write me at quadtree@<domain>.

8 References

For the images, I've used colors from the tomorrow-light variant of the base16 color scheme. Thanks Chris Kempson!